Assessment 2x10=20

Topic: Compact Metric Spaces

Choose the correct option

1. If A is a closed subset of a compact metric space (X, d), then

(a) A may not be compact (b) A is compact and bounded

(c) A is compact but not bounded (d) none of these

2. A metric space (X, d) is sequentially compact, then the statements

P: Every sequence $\{x_n\}$ from X have convergent subsequence.

Q: (X, d) is complete and totally bounded.

(a) P is true, Q is false (b) P is false, Q is true

(c) Both P and Q are false (d) both P and Q are true

3. The set $X = \mathbb{R}$ with the metric $d(x, y) = \frac{|x-y|}{1+|x-y|}$ is

(a) Bounded but not compact (b) bounded but not complete

(c) Complete but not bounded (d) compact but not complete

4. Let$X = (0,2]$. In (X, d_u) consider $A = (0,1]$, then

(a) A is closed but not compact (b) A is open and compact

(c) A is closed and compact (d) A is open but not compact

5. Which of the following is not totally bounded?

$(a)\ A = \{x \in \mathbb{R}: 1 \leq x \leq 5\}$ $(b) A = \{x \in \mathbb{R}: -5 < x < 2\}$

$(c)\ A = \left\{x \in l_2: \sqrt{\sum_{i=1}^{\infty} {x_i}^2}\right\} = 1$ $(d) A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq 1\}$

6. Let (X, d) and (Y, ρ) be metric spaces and $f: X \to Y$ be a map, then which of the following statements is false?

(a) If f is continuous and $A(\subseteq X)$ is compact, then $f(A)$ is closed and bounded in Y

(b) If f is continuous and X is compact and $A(\subseteq X)$ is closed in X, then $f(A)$ is closed and bounded in Y

(c) If $A(\subseteq X)$ is compact and $f(A)$ is compact, then f is continuous

(d) If f is continuous and $A(\subseteq X)$ is compact then $f(A)$ is compact

7. A metric space (X, d) satisfies Bolzano Weierstrass Property then

(a) Every infinite sequence $\{x_n\}$ has no cluster point

(b) Every infinite sequence $\{x_n\}$ has at least one cluster point

(c) (X, d) is not sequentially compact

(d) (X, d) is not compact

8. Which of the following subset of $\mathbb{R}$ is closed but not compact

$(a)\ \mathbb{N}$ $(b)\ [-2,5]$

$(c)\ \{-8,-1,5,7\}$ $(d)\ Z \cap (-10,10)$

9. Let (X,d) be a metric space. Consider the following statements:

P: $A \subseteq X$ is compact $\Rightarrow A$ is totally bounded
Q: $A \subseteq X$ is totally bounded $\Rightarrow A$ is compact
R: $A \subseteq X$ is bounded $\Rightarrow A$ is totally bounded
S: $A \subseteq X$ is totally bounded $\Rightarrow A$ is bounded, then

$(a)\ only\ P\ and\ Q\ are\ true$
$(b)\ only\ Q\ and\ R\ are\ true$
$(c)\ only\ P\ and\ S\ are\ true$
$(d)\ P, Q, R, S\ all\ are\ true$

10. If F is a closed and bounded subset of $\mathbb{R}$, then each open covering of F have

(a) Finite sub-covering of F

(b) No finite sub-covering of F

(c) Infinite sub-covering of F

(d) None of the above